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Astrophysical Effects of $\nu\gamma \to \nu\gamma\gamma$ and Its Crossed Processes

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Abstract

Recently, Dicus and Repko computed $\nu\gamma \to \nu\gamma\gamma$ for energies below the threshold for e^+e^- pair production. They found a cross section on the order of $10^{-52}\omega^{\gamma}$ with $\gamma = 10$, where ω is the CMS energy of one of the initial particles in MeV. Cross sections for the crossed processes are the same to factors of order one. This note investigates the extent to which these processes could, if their result extrapolates past 1 MeV: affect supernova dynamics; cut off the energy distribution of very high energy cosmic photons and neutrinos; and possibly give rise to an observable gamma signal from scattering of neutrinos from one supernova by those of a second supernova close in space and time. We also estimate, from Supernova 1987A, that, in the region above a few MeV, γ must fall below 8.4.

The process $\nu\gamma \to \nu\gamma$ is quite small (about 10^{-65} at 1 MeV) because Yang's theorem [1] (two photon decay of spin 1 particles of either parity forbidden) causes it to vanish in the limit of zero momentum transfer (in lowest order). The reaction $\nu\gamma \to \nu\gamma\gamma$ and its crossed processes are not subject to this suppression [2,3] and hence are potentially much more important to astrophysics. The result of reference 3 is

$$\sigma = \sigma_0 \left(\frac{\omega}{1\ MeV}\right)^{\gamma} \qquad \sigma_0 = 10^{-52} cm^2 \qquad \gamma = 10 \tag{1}$$

for $\omega < 0.5$ MeV. That paper speculates that $Eq.$ (1) remains valid for at least another order of magnitude in ω. However, due to the complexity of the process, no evaluation of the cross section above the pair production threshold has been done. The purpose of this note is, in part, to investigate the extent to which such evaluation might be motivated by astrophysical applications – particularly supernova dynamics and attenuation of very high energy neutrinos and photons from scattering off photon and neutrino backgrounds. We thus assume at first that $Eq.$ (1) is valid for ω in the region of tens of MeV's [1].

The family of related processes

$$\nu\gamma \to \nu\gamma\gamma \ \ (2a) \qquad \nu\bar{\nu} \to \gamma\gamma\gamma \ \ (2b) \qquad \gamma\gamma \to \nu\bar{\nu}\gamma \ \ (2c) \tag{2}$$

has several features of interest: all five particles are essentially massless (CMS electron neutrino energies will be far above the 10 eV extreme limit on its mass). Thus the three processes have essentially the same cross section up to factors associated with spin and statistics. Processes $(2a)$ and $(2b)$ are potentially important contributions to supernova neutrino mean free paths. They also contribute to ultra high energy photon scattering off cosmic background neutrinos as well as to ultra high energy scattering of neutrinos off

[1]We note for comparison that, if the cross section of Eq. (1) were limited to a small number of partial waves, the partial wave unitarity limit would be exceeded for ω greater than about 300 MeV.

various cosmic photon backgrounds. Process ($2c$) is a possible energy loss mechanism for hot objects; it could be important in such contexts as the cooling of neutron stars.

We label the initial particles q_1 and q_2, the final ones p_1, p_2, p_3, and the number of spin states $g(q_i)$ and $g(p_i)$. The rate for interaction is then given by [4]

$$\frac{dn_{12}}{dt} = (2\pi)^{-11} 2^{-5} g_i g_f \int\int \prod_{i=1}^{2} \frac{d^3 q_i}{q_i} \prod_{j=1}^{3} \frac{d^3 p_i}{p_i} f(q_1) f(q_2) \prod_{k=1}^{3} (1 \pm f(p_k)) \\ \delta^4[q_1 + q_2 - p_1 - p_2 - p_3] \mid M \mid^2 \tag{3}$$

where

$$g_i = g(q_1)g(q_2) \qquad g_f = g(p_1)g(p_2)g(p_3)$$

for processes ($2a$), ($2b$) and ($2c$) respectively, and the f's are bose and fermi distributions for γ and ν. We will consider the possibility that $T_\gamma \neq T_\nu$. We ignore the factors of g as well as the angular dependence of M. From $Eq.$ (1) we have, after evaluating the CMS phase space integral

$$\mid M \mid^2 = 2^6 (2\pi)^3 \sigma_0 \omega^\gamma \tag{4}$$

The rate for process (2a) then becomes

$$\frac{dn_{\nu\gamma}}{dt} = \frac{2\sigma_0}{(2\pi)^8} \int \frac{d^3 q_1}{q_1} f_F(q_1) \frac{d^3 q_2}{q_2} f_B(q_2) \frac{d^3 p_1}{p_1} (1 - f_F(p_1)) \frac{d^3 p_2}{p_2} (1 + f_B(p_2)) \\ \frac{d^3 p_3}{p_3} (1 + f_B(p_3)) \omega^\gamma \delta^4[q_1 + q_2 - p_1 - p_2 - p_3] \tag{5}$$

where ${\omega_{cm}}^2 = q_1 \cdot q_2/2$. We eliminate the q_2 integral with the $3-D$ delta function and write for the time component

$$\delta^4[q_2 - (p_1 + p_2 + p_3 - q_1)] = \quad q_2 \delta\{Dp_3 - [(p_1 + p_2)^2/2 - q_1(p_1 + p_2) + \mathbf{q_1} \cdot (\mathbf{p_1} + \mathbf{p_2})]\} \tag{6}$$

where

$$D = q_1 - p_1 - p_2 + (\mathbf{p_1} + \mathbf{p_2}) \cdot \hat{\mathbf{p}}_\mathbf{3} - \mathbf{q_1} \cdot \hat{\mathbf{p}}_\mathbf{3}$$

with $\hat{\mathbf{p}}_3$ the unit vector in the direction of $\mathbf{p_3}$. This gives

$$\frac{dn_{12}}{dt} = \frac{4\sigma_0}{(2\pi)^6} \int dq_1 dp_1 dp_2 dz_1 dz_2 dz_3 d\phi_1 d\phi_2 q_1 (p_1 p_2 p_3) D^{-1} \\ f_F(q_1) f_B(q_2) \prod_{i=1}^{3} (1 \pm f(p_i)) \Theta(q_2) {\omega_{cm}}^\gamma \tag{7}$$

where we have taken q_1 along the z-axis and p_3 in the $x - z$ plane with no loss of generality. The expressions for processes (2b) and (2c) are the same except for the fermi and bose factors. From Eqs. (7) and (8), we can find the mean free path, λ_1, for particle 1 by

$$\lambda_1 = cn_1/(\frac{dn_{12}}{dt}) \tag{8}$$

We evaluated the integral of $Eq.$ (7) for various cases with Monte Carlo methods: First, if $T_\nu = T_\gamma = T$ and $\mu_\nu = 0$, one sees from dimensional analysis of Equations (7) and (8) that λ_1 must go as $T^{-(\gamma+3)}$. Thus we can write

$$\lambda_1 = \Lambda T^{-13} cm \qquad (9)$$

The Monte Carlo integration gives $\Lambda \sim 2.2 \times 10^{14} cmMeV^{13}$ for $\gamma = 10$. T is measured in MeV and the result is essentially the same for all three processes. *Table* 1 gives results for several choices of T and non-zero values of μ_ν ($\mu_{\bar{\nu}} = -\mu_\nu$). The degeneracy parameter $\eta = \mu/T$ can be found from the values given. The important feature is that for $\omega \geq 5$ MeV, the mean free path for a neutrino is less than the size, 10^6 cm, of the collapsing core. *Table* 2 gives results for several choices of T_ν and μ_ν holding T_γ fixed at 5 MeV. Note from the tables that the effect of final state Bose-Einstein enhancement for ($2b$) becomes dramatic for $T_\nu > T_\gamma$. No results for ($2c$) are given in *Table* 2 because the variation is less than a factor of ten over all the values considered [2].

We now turn to the implications of these results for supernova dynamics [5]. Since the mean free paths for the processes of *Eq.* (2) are less than the size of the supernova core (10^6 cm) for much of the parameter space, it would appear desirable that these processes be included in supernova codes. Some of the steps in which they might be particularly important include: (i) The collapse phase. During collapse, the effect of $\nu_e - e$ scattering is to lower neutrino energies facilitating their escape and thereby to decrease the electron fraction which, in turn, results in bounce from a smaller core and hence a smaller shock to drive a larger mass [5,6]. The results of *Table* 2 imply that process ($2a$) could play a similar role for ω over a few MeV and that therefore prompt shock mechanisms are only feasible if photon temperatures during infall are kept below about 5 MeV. (ii) Immediately after bounce. At this time, the hot core has temperatures on the order of $20 - 40$ MeV and neutrino energies range beyond 100 MeV. At these energies, the validity of *Eq.* (1) is more suspect, but *Table* 1 shows that even a much more gentle rise with temperature would have short mean free paths for the processes of *Eq.* (2). (iii) The first second. So long as T stays above a few MeV, the processes of *Eq.* (2) should remain important.

Finally, we note that a bound can be placed on the cross section for ($2a$) from the observed distribution of neutrinos from Supernova 1987A. We can ask whether *Eq.* (1) is consistent with the fitting parameters of roughly 4 seconds of neutrino radiation at a temperature of 4 MeV [5]. A neutrino of about 1 MeV or less leaves the supernova immediately without further scattering, so that we need the mean free path for (2a) with the final neutrino having energy less than 1 MeV to be greater than 10^{11} cm. *Eq.* (1) gives only 5×10^8 cm (when the appropriate theta function is inserted into *Eq.* (7)). Replacing $\gamma = 10$ in *Eq.*(1) by $\gamma = 8.4$ (for $E > 1$ MeV) changes the result to 10^{11} cm [10].

A second area in which the processes of *Eq.* (2) are potentially of interest is that of long distance travel of extremely energetic neutrinos and photons given the existence of the cosmic microwave gamma and neutrino backgrounds as well as IR and optical backgrounds [7]. They will attenuate the high energy particle over cosmological distances, L, for energies such that

$$\int \frac{dn_B}{dE_B} \sigma(E, E_B) dE_B L > 1 \qquad (10)$$

where E and E_B refer to the energetic and background particles respectively. Suppose, for example, ν_e had a mass of 10 eV. *Eq.* ($2a$) would give L approximately 10^9 lys at $E = 4 \times 10^{16}$ eV, corresponding to $\omega \sim 0.3$ GeV, were *Eq.* (1) to be trusted that far. Using

[2] We have estimated relativistic plasma effects by evaluating the integrals with integrands set to zero for photon energies less than the plasma frequency as given in, for example, E. Braaten and D. Segel, Phys. Rev. D48, 1478 (1993). The result is increase in mean free paths of less than 10 percent.

instead $\gamma = 8.4$ would give $E = 4 \times 10^{17}$ eV, corresponding to $\omega \sim 1$ GeV. In *Table* (3), we list results for a few other cases of interest.

An amusing application of $(2b)$ is the case of two supernovae close to each other in space and time. Massive stars formed in giant molecular clouds will be separated by distances on the order of 100 light years and 100 years of each other and will have lifetimes on the order of a million years. The chances of two type II supernova explosions in one cloud within 100 lys are perhaps non-negligible. Gamma rays produced at the intersection of the two expanding neutrinospheres would, like the radiation from the supernovae themselves [8], be absorbed by the cloud, but the signal from the re-radiated IR could, in principle, be of interest. The total energy in the signal would be on the order of

$$E_T = \int dE_{\bar{\nu}} dE_\nu \frac{dN(E_\nu)}{dE_\nu}(E_\nu + \bar{E}_\nu)\frac{dn(E_{\bar{\nu}})}{dE_{\bar{\nu}}}\sigma c \Delta t \sim 10^{29} ergs \tag{11}$$

which, when spread over ten years is unfortunately far below the diffuse IR background. For completeness, we also note that, in a cooling neutron star, one can estimate from *Eq.* (10) that *Eq.* $(2c)$ should go roughly as $10^{31} T_9^{17} erg/s$ which falls below the URCA process [9] $(5 \times 10^{39} T_9^8)$ for temperatures under about an MeV.

In conclusion, the processes of *Eq.* (2) could be quite important in astrophysics, particularly in supernova dynamics and long distance travel of photons and neutrinos. From Supernova 1987A, we can deduce that *Eq.* (1) must be modified in the region beyond an MeV or so by, at least, reduction of the exponent γ from 10 to 8.4. Clearly a calculation of the processes of *Eq.* (2) in the region above the pair production threshhold would be quite useful.

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TABLES

Table 1. Mean free paths for the three processes of *Eq.* (2) for the values of temperature and chemical potential indicated. Asterisks indicate MFP's greater than a light year.

$\nu\gamma \to \nu\gamma\gamma$			Mean Free Path(cm)					μ_ν
3.0×10^{14}	2.4×10^{10}	1.5×10^{5}	2.0×10^{1}	1.2×10^{-1}	2.9×10^{-3}	1.8×10^{-8}	2.2×10^{-12}	-50
2.1×10^{14}	2.8×10^{10}	1.6×10^{5}	2.3×10^{1}	1.2×10^{-1}	2.8×10^{-3}	1.9×10^{-8}	2.3×10^{-12}	-30
2.1×10^{14}	2.6×10^{10}	1.9×10^{5}	2.3×10^{1}	1.2×10^{-1}	2.8×10^{-3}	1.8×10^{-8}	2.2×10^{-12}	-20
2.0×10^{14}	2.4×10^{10}	1.9×10^{5}	2.3×10^{1}	1.1×10^{-1}	2.8×10^{-3}	1.9×10^{-8}	2.2×10^{-12}	-10
2.2×10^{14}	2.7×10^{10}	1.8×10^{5}	2.2×10^{1}	1.1×10^{-1}	2.7×10^{-3}	1.8×10^{-8}	2.1×10^{-12}	0
8.7×10^{12}	1.0×10^{10}	1.5×10^{5}	2.0×10^{1}	1.1×10^{-1}	2.6×10^{-3}	1.8×10^{-8}	2.2×10^{-12}	10
3.4×10^{13}	4.4×10^{9}	7.4×10^{4}	1.7×10^{1}	9.5×10^{-2}	2.4×10^{-3}	1.8×10^{-8}	2.2×10^{-12}	25
1.3×10^{13}	1.0×10^{9}	3.0×10^{4}	9.0×10^{0}	7.4×10^{-2}	2.1×10^{-3}	1.7×10^{-8}	2.1×10^{-12}	50
$\nu\nu \to \gamma\gamma\gamma$								
5.2×10^{5}	1.0×10^{4}	5.6×10^{1}	3.2×10^{-1}	8.7×10^{-3}	4.6×10^{-4}	1.3×10^{-8}	2.5×10^{-12}	-50
2.2×10^{7}	5.4×10^{5}	1.1×10^{3}	2.3×10^{0}	3.1×10^{-2}	1.2×10^{-3}	1.9×10^{-8}	3.1×10^{-12}	-30
5.3×10^{8}	8.9×10^{6}	7.6×10^{3}	6.0×10^{0}	5.8×10^{-2}	1.9×10^{-3}	2.2×10^{-8}	3.3×10^{-12}	-20
6.9×10^{10}	3.9×10^{8}	4.9×10^{4}	1.6×10^{1}	1.1×10^{-1}	3.1×10^{-3}	2.8×10^{-8}	3.5×10^{-12}	-10
4.0×10^{14}	4.9×10^{10}	3.2×10^{5}	4.0×10^{1}	2.0×10^{-1}	4.9×10^{-3}	3.3×10^{-8}	3.9×10^{-12}	0
2.5×10^{17}	1.6×10^{12}	1.7×10^{6}	9.5×10^{1}	3.7×10^{-1}	7.5×10^{-3}	3.9×10^{-8}	4.3×10^{-12}	10
***	2.0×10^{14}	1.1×10^{7}	3.1×10^{2}	8.3×10^{-1}	1.5×10^{-2}	5.1×10^{-8}	5.0×10^{-12}	25
***	***	2.2×10^{8}	1.4×10^{3}	2.8×10^{0}	3.9×10^{-2}	7.9×10^{-8}	6.3×10^{-12}	50
$\gamma\gamma \to \gamma\nu\nu$								
***	***	2.3×10^{6}	5.1×10^{1}	2.2×10^{-1}	4.7×10^{-3}	2.7×10^{-8}	3.2×10^{-12}	-50
***	6.0×10^{12}	5.3×10^{5}	4.0×10^{1}	1.9×10^{-1}	4.3×10^{-3}	2.8×10^{-8}	3.3×10^{-12}	-30
***	3.5×10^{11}	4.0×10^{5}	3.6×10^{1}	1.8×10^{-1}	4.1×10^{-3}	2.6×10^{-8}	3.2×10^{-12}	-20
3.1×10^{15}	6.1×10^{10}	2.9×10^{5}	3.3×10^{1}	1.7×10^{-1}	4.0×10^{-3}	2.7×10^{-8}	3.2×10^{-12}	-10
3.3×10^{14}	4.0×10^{10}	2.7×10^{5}	3.2×10^{1}	1.7×10^{-1}	3.9×10^{-3}	2.7×10^{-8}	3.2×10^{-12}	0
2.9×10^{15}	5.9×10^{10}	2.9×10^{5}	3.3×10^{1}	1.7×10^{-1}	4.0×10^{-3}	2.7×10^{-8}	3.2×10^{-12}	10
***	1.3×10^{12}	4.2×10^{5}	3.8×10^{1}	1.7×10^{-1}	4.2×10^{-3}	2.7×10^{-8}	3.2×10^{-12}	25
***	***	2.2×10^{6}	5.0×10^{1}	2.2×10^{-1}	4.7×10^{-3}	2.8×10^{-8}	3.3×10^{-12}	50
$T = 1$	2	5	10	15	20	50	100 MeV	

Table 2. Mean free paths for the processes of *Eq.* (2*a*) and (2*b*) with the photon temperature held constant at $5 MeV$. Process (2*c*) varies little and is omitted.

$\nu\gamma \to \nu\gamma\gamma$			Mean Free Path(cm)					μ_ν
1.8×10^{5}	5.8×10^{3}	7.5×10^{2}	2.0×10^{2}	7.2×10^{1}	2.6×10^{1}	1.3×10^{1}	6.3×10^{0}	0
1.5×10^{5}	5.4×10^{3}	7.6×10^{2}	1.8×10^{2}	6.9×10^{1}	2.4×10^{1}	1.2×10^{1}	7.3×10^{0}	10
1.1×10^{5}	5.4×10^{3}	7.3×10^{2}	2.0×10^{2}	6.3×10^{1}	3.0×10^{1}	1.3×10^{1}	4.7×10^{0}	20
7.4×10^{4}	4.7×10^{3}	7.9×10^{2}	1.6×10^{2}	6.3×10^{1}	2.5×10^{1}	1.3×10^{1}	9.9×10^{0}	25
5.5×10^{4}	5.1×10^{3}	8.3×10^{2}	1.5×10^{2}	6.0×10^{1}	2.3×10^{1}	1.2×10^{1}	7.9×10^{0}	30
4.7×10^{4}	4.3×10^{3}	7.5×10^{2}	1.9×10^{2}	6.1×10^{1}	2.3×10^{1}	1.3×10^{1}	5.8×10^{0}	35
4.1×10^{4}	4.5×10^{3}	7.1×10^{2}	2.0×10^{2}	6.7×10^{1}	2.6×10^{1}	1.2×10^{1}	8.3×10^{0}	40
3.8×10^{4}	4.0×10^{3}	7.0×10^{2}	1.9×10^{2}	6.1×10^{1}	2.4×10^{1}	1.7×10^{1}	7.9×10^{0}	45
$\nu\nu \to \gamma\gamma\gamma$								
3.2×10^{5}	4.7×10^{1}	2.5×10^{-1}	6.1×10^{-3}	3.3×10^{-4}	3.9×10^{-5}	7.0×10^{-6}	1.9×10^{-6}	0
1.7×10^{6}	1.1×10^{2}	4.5×10^{-1}	9.2×10^{-3}	4.8×10^{-4}	5.2×10^{-5}	9.5×10^{-6}	2.4×10^{-6}	10
7.3×10^{6}	2.5×10^{2}	8.0×10^{-1}	1.4×10^{-2}	6.7×10^{-4}	7.0×10^{-5}	1.2×10^{-5}	2.9×10^{-6}	20
1.1×10^{7}	3.6×10^{2}	1.0×10^{0}	1.8×10^{-2}	7.9×10^{-4}	8.0×10^{-5}	1.4×10^{-5}	3.1×10^{-6}	25
1.9×10^{7}	5.2×10^{2}	1.3×10^{0}	2.2×10^{-2}	9.4×10^{-4}	9.1×10^{-5}	1.5×10^{-5}	3.6×10^{-6}	30
3.4×10^{7}	6.9×10^{2}	1.7×10^{0}	2.6×10^{-2}	1.1×10^{-3}	1.0×10^{-4}	1.7×10^{-5}	3.9×10^{-6}	35
6.9×10^{7}	9.9×10^{2}	2.2×10^{0}	3.2×10^{-2}	1.3×10^{-3}	1.2×10^{-4}	1.9×10^{-5}	4.4×10^{-6}	40
1.3×10^{8}	1.2×10^{3}	2.8×10^{0}	3.8×10^{-2}	1.5×10^{-3}	1.4×10^{-4}	2.2×10^{-5}	5.1×10^{-6}	45
$T = 1$	2	5	10	15	20	50	100 MeV	

Table 3. Energies at which mean free paths through cosmic backgrounds drop below 10^9 light years, for both $\gamma = 10$ and $\gamma = 8.4$ in $Eq.(1)$.

Particle	Background	Energy at $\lambda = 10^9$ lys $\gamma = 10$	$\gamma = 8.4$
Neutrino	Photon$(E = 2 \times 10^{-4} eV)$	$2 \times 10^{20} eV$	$1 \times 10^{22} eV$
Photon	Neutrino$(m = 10eV)$	$4 \times 10^{16} eV$	$4 \times 10^{17} eV$
Neutrino	IR Photon$(E = 4 \times 10^{-3} - 0.1eV)$	$2 \times 10^{16} eV$	$4 \times 10^{17} eV$
Neutrino	Starlight Photon$(E = 0.1 - 10eV)$	$1 \times 10^{15} eV$	$5 \times 10^{16} eV$